Quiz 4A, Calculus I

Dr. Graham-Squire, Spring 2013

1:16

1:22

6 ⟹ give 20 min.

Name: Key

1. (3 points) Use logarithmic differentiation to find $f'(x)$ for $f(x) = x^{(\cos x)}$.

$$y = x^{\cos x}$$

$$\ln y = \ln x^{\cos x}$$

$$\frac{d}{dx}\left(\ln y = (\cos x)(\ln x) \right)$$

$$\frac{y'}{y} = -\sin x \ln x + \frac{\cos x}{x}$$

$$\boxed{y' = x^{(\cos x)}\left(\frac{\cos x}{x} - (\sin x)(\ln x) \right)}$$

2. (3 points) Use differentials to approximate $\sqrt{4.1}$.

$\sqrt{4.1} \approx \sqrt{4} = 2$

$f(x) = \sqrt{x}$ $\quad s = 4$

$f'(x) = \frac{1}{2} x^{-1/2}$ $\quad dx = 0.1$

$f'(x) = \frac{1}{2\sqrt{x}}$

$dy = f'(s)\,dx$

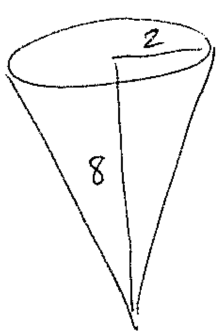
$dy = \left(\frac{1}{2\sqrt{4}}\right)(0.1)$

$= \frac{1}{4}(0.1)$

$dy = 0.025$

$\Rightarrow \sqrt{4.1} \approx 2 + 0.025 = 2.025$

3. (4 points) Water is leaking out of an inverted conical tank at a rate of 2 ft^3/min. The tank has a height of 8 feet and a radius at the top of 2 feet. How fast is the height of the water level changing when the water level is 3 feet high? The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.



$\cancel{\frac{d}{dt}\left(V = \frac{1}{3}\pi r^2 h\right)}$

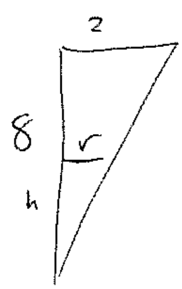
$\frac{dV}{dt} = -2$

WTF: $\left.\frac{dh}{dt}\right|_{h=3}$

$\Rightarrow V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{4}\right)^2 h$

$\frac{d}{dt}\left(V = \frac{\pi}{3} \cdot \frac{h^3}{16}\right)$



$\frac{dV}{dt} = \frac{\pi}{\cancel{3}} \cdot \frac{1}{16} \cdot \cancel{3} h^2 \frac{dh}{dt}$

$\frac{2}{8} = \frac{r}{h}$

$\frac{dV}{dt} = \frac{\pi}{16} h^2 \cdot \frac{dh}{dt}$

$r = \frac{h}{4}$

$-2 = \frac{\pi}{16}(3)^2 \frac{dh}{dt}$

$-1.13 \approx \frac{-32}{9\pi} = \frac{dh}{dt}$ ft/min

$\boxed{\text{water level is dropping at a rate of } 1.13 \text{ ft/min}}$

Quiz 4B, Calculus I

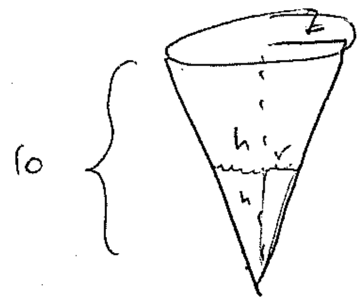
Dr. Graham-Squire, Spring 2013

Name: Key

1. (4 points) Water is leaking out of an inverted conical tank at a rate of 3 ft^3/min. The tank has a height of 10 feet and a radius at the top of 2 feet. How fast is the height of the water level changing when the water level is 4 feet high? The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Round your answer to the nearest 0.01.

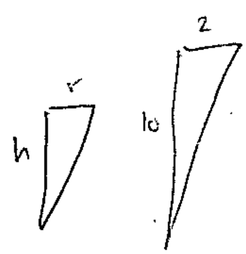
$$\frac{dV}{dt} = -3 \text{ ft}^3/\text{min}$$

Want $\left.\frac{dh}{dt}\right|_{h=4}$.



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{\pi}{3}\left(\frac{h}{5}\right)^2 \cdot h$$



$$\frac{d}{dt}\left(V = \frac{\pi}{3} \cdot \frac{1}{25} \cdot h^3 \right)$$

$$\Rightarrow \frac{r}{h} = \frac{2}{10} = \frac{1}{5}$$

$$r = \frac{h}{5}$$

$$\frac{dV}{dt} = \frac{\pi}{75} \cdot 3h^2 \frac{dh}{dt}$$

$$\Rightarrow -3 = \frac{\pi}{25} \cdot 4^2 \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{-3 \cdot 25}{16\pi} = \frac{dh}{dt}$$

$$-1.49 = \frac{dh}{dt}$$

Water height is dropping at 1.49 ft/min

2. (3 points) Use logarithmic differentiation to find $f'(x)$ for $f(x) = x^{(\sin x)}$.

$$y = x^{\sin x}$$

$$\ln y = \ln x^{\sin x}$$

$$\frac{d}{dx}\left(\ln y = (\sin x)(\ln x) \right)$$

$$\frac{y'}{y} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$$

$$\boxed{y' = x^{\sin x}\left((\cos x)\ln x + \frac{\sin x}{x} \right)}$$

3. (3 points) Use differentials to approximate $\sqrt{8.9}$. → almost 3, slightly less

$s = 9$

$dx = -0.1$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2}x^{-1/2}$

$$dy = f'(s)\,dx$$

$$\Rightarrow dy = \frac{1}{2}(9)^{-1/2} \cdot (-0.1)$$

$$dy = \frac{1}{2} \cdot \frac{1}{3} \cdot (-0.1)$$

$$= \frac{-1}{60} \approx -0.01\overline{666}$$

$$\Rightarrow \sqrt{8.9} \approx 3 - 0.01\overline{6} = \boxed{2.98\overline{3}}$$